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Fracture Criterion for Notched Thin Composite Laminates

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Fracture behavior of fiber-dominated center-notched AS4/3501-6 graphite-epoxy laminates is investigated in this study. Nine laminate configurations are studied to examine flaw size effects, crack tip damage mechanisms, and failure modes under uniaxial tensile loading. Results indicate that a constant value of fracture toughness K_Q is a laminate material property. A layup independent failure criterion is proposed, which relates laminate fracture toughness to the fracture toughness K_Q^0 of the principal load bearing ply. K_Q^0 characterizes the in situ fracture toughness of a notched 0-deg layer in the event of fiber breakage along the plane of the notch. Once its value is estimated from preliminary tests, this parameter can be used to predict fracture toughness, and hence residual strength, of other fiber-dominated laminates of the same material system. The model predictions agree well with current experimental results, as well as with data published by other researchers. The model is further extended to predict residual strength of laminates with inclined cracks (mixed-mode loading). It is demonstrated that the normal projection of the crack to the applied load can be considered as the equivalent crack and governs laminate residual strength.

Introduction

COMPOSITE materials are extremely notch sensitive. The presence of cracks in structural components drastically reduces their load carrying capacity. For this reason, the issue of predicting composite residual strength in the presence of stress enhancers such as cracks has been an important research problem in the composites community.

A vast amount of experimental literature is available on the notched strength behavior of different composite systems. Several strength-based and fracture mechanics-based models have been proposed in the literature to predict the notched response of composite laminates under uniaxial tensile loading. The popular models, e.g., those by Whitney and Nuismer,[1] Waddoups et al.,[2] Pipes et al.,[3] and Karlak,[4] etc., are based on a characteristic distance concept. In these studies, the damage events are assumed to occur in a region a_0 ahead of the crack tip. The value for a_0 is then chosen to fit the experimental data. Such models have been successfully used to predict laminate strength in the presence of notches and holes. A comprehensive review of these models has been done by Awerbuch and Madhukar.[5]

One limitation of such an approach is that a_0 is not a material constant and depends on factors such as notch size and laminate orientation.[5] As such, results obtained from tests on one laminate configuration cannot be extrapolated to predict the fracture toughness of other laminates of the same material system.

In this study, an alternative approach to predicting notched strength of a class of laminates is presented. The goal is to develop a means to predict fracture of a class of fiber-dominated laminates without having to resort to experimental determination of the characteristic distance for each laminate configuration. Past experimental investigations[5–9] have revealed that fiber breakage in the principal load bearing plies precipitates laminate failure. Hence, studying when and how these plies fail could lead to a means of developing a layup independent failure criterion. Such an approach has been adopted by Kageyama[7] and Kortschot and Beaumont[8] in studying the failure of cross-ply laminates. In this paper, the concept introduced by Kageyama[7] is extended to other laminate configurations, and a simple model is developed to determine the parameter that governs failure of the principal load bearing plies.

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Experiments were conducted to examine the notched behavior of different fiber-dominated laminates. The aims of the present research were the following: 1) study crack tip damage mechanisms and failure modes in center notched composite laminates, 2) determine if fracture mechanics concepts such as critical stress intensity factor or fracture toughness could be used to describe notched behavior of these composites, and 3) develop a layup independent failure model for predicting notched composite strength.

Experimental Procedure

The material system chosen for the study was AS4/3501-6 graphite epoxy manufactured by Hercules, Inc. The material is available in the form of a unidirectional prepreg tape with a nominal thickness of 0.127 mm. The unidirectional material properties are reported in Table 1. The laminates were made using the hand layup technique and cured in an autoclave. Nine different laminate configurations (Table 2) were chosen such that they had at least some 0-deg plies, i.e., plies with fibers aligned along the loading direction.

Test specimens were cut from 30.5 × 30.5 cm panels on a FLOW® waterjet system. The test configuration was that of a center crack oriented perpendicular to the loading direction. The specimen dimensions were 25.4 × 3.81 cm with 3.81-cm-long end tabs. To make the cracks, a starter hole was first drilled in the laminate to minimize any delamination caused by the waterjet. The crack was then made

Table 1 Material properties of unidirectional AS4/3501-6 graphite epoxy

E_1, GPa	E_2, GPa	G_{12}, GPa	G_{13}, GPa	G_{23}, GPa	ν_{12}	ν_{13}	ν_{23}
138	9.65	5.24	5.24	3.24	0.3	0.3	0.49

Table 2 Fracture toughness of AS4/3501-6 graphite epoxy laminates

Laminate configuration	Fracture toughness K_Q, MPa $\sqrt{\text{m}}$	Stress ratio η	Fracture toughness of 0-deg ply K_Q^0, MPa $\sqrt{\text{m}}$
$[0/90/\pm 45]_s$	44.83 ± 3.05	2.58	115.66
$[\pm 45/\overline{90}/0]_s$	40.28 ± 1.87	2.58	104.43
$[\overline{90}/0/\pm 45]_s$	41.25 ± 1.46	2.58	106.43
$[0/\pm 15]_s$	90.82 ± 4.01	1.12	101.81
$[0/\pm 30]_s$	60.84 ± 3.14	1.66	101
$[0/\pm 45]_s$	45.83 ± 2.17	2.32	106.32
$[0/\overline{90}]_{2s}$	58.31 ± 5.30	1.87	109.04
$[\pm 45/0/\pm 45]_s$	37.48 ± 0.43	3.2	119.81
$[\pm 45_2/0/\pm 45]_s$	32.42 ± 0.6	3.82	123.64

by a waterjet cut and further extended with a 0.2-mm-thick jeweler's saw blade.

Quasistatic experiments were conducted in a servohydraulically driven MTS machine with an Instron 8500 controller under position control at a head displacement rate of 0.254 mm/min. The load and head displacement history were recorded using LabVIEW software.

Damage Mechanisms and Failure Modes

An important objective of the experiments was to monitor damage growth and failure mechanisms ahead of the crack tip. The test specimens were examined prior to testing to ensure that no significant damage occurred during their preparation. Damage evolution was studied by diiodobutane enhanced x-ray examination at different load levels. The fractured specimens were also examined to determine the failure mode of the individual plies.

All notched laminates exhibited a linear load vs displacement relationship up to failure. Such behavior is typical of fiber-dominated laminates. Figure 1 shows typical crack tip damage in a $[0/90/\pm 45]_s$ quasi-isotropic laminate. The damage zone in such laminates is characterized by matrix cracks in off-axis plies, axial splits (matrix cracks along the fiber direction of the 0-deg ply), and some delamination. The matrix cracks are observed as dark lines in the figures. (The dark region in the center part of the crack is cutting tool induced damage and not load induced and, hence, should be ignored. It does not appear to affect the notched strength of the laminate.)

Radiographs of $[\pm 45/0/\pm 45]_s$ and $[\pm 45_2/0/\pm 45]_s$ laminates (Figs. 2 and 3) reveal similar matrix cracks in the off-axis plies along with some delamination. Inspection of the failed specimens revealed that the off-axis plies failed along matrix cracks extending from the crack tip to the specimen edge, whereas the 0-deg plies failed by fiber breakage.

The $[0/\pm\theta]_s$ laminates (Figs. 4–6) show matrix cracks in the off-axis plies, delamination at the $[\pm\theta]$ interface, and axial splits in the 0-deg plies. The 0-deg plies in these laminates exhibited fiber breakage while the other plies failed along matrix cracks.

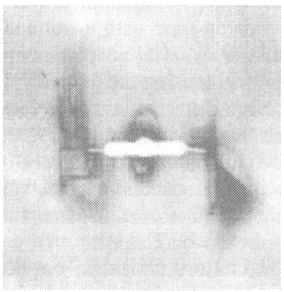

Fig. 1 Laminate $[0/90/\pm 45]_s$ at 99% of failure load.

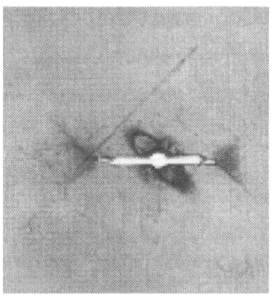

Fig. 2 Laminate $[\pm 45/0/\pm 45]_s$ at 95% of failure load.

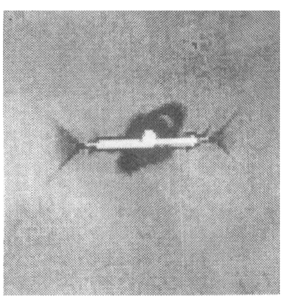

Fig. 3 Laminate $[\pm 45_2/0/\pm 45]_s$ at 97% of failure load.

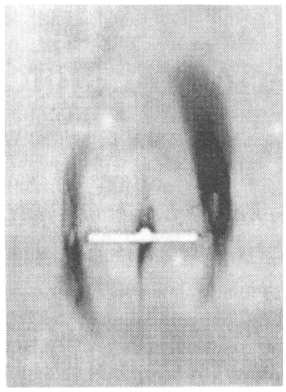

Fig. 4 Laminate $[0/15/\pm 15]_s$ at 92% of failure load.

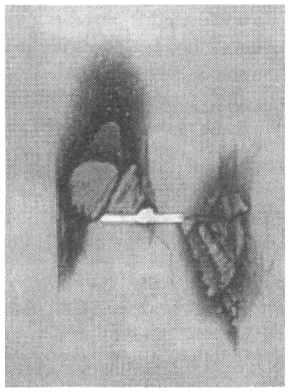

Fig. 5 Laminate $[0/30/\pm 30]_s$ at 99% of failure load.

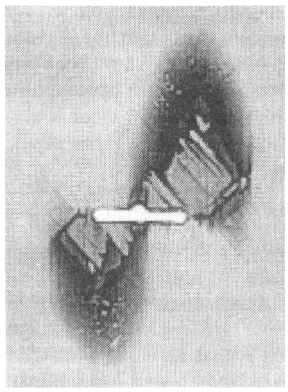

Fig. 6 Laminate $[0/45/\pm 45]_s$ at 99% of failure load.

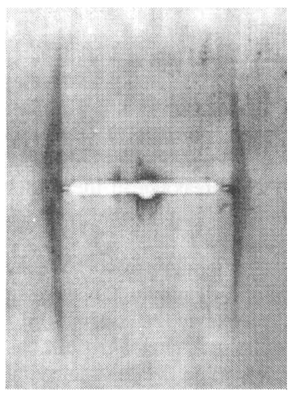

Fig. 7 Laminate $[0/90]_{2s}$ at 85% of failure load.

A study of cross-ply $[0/90]_{2s}$ laminates shows long axial splits emerging from the notch tips of the 0-deg plies (Fig. 7). Transverse matrix cracks are also observed in the 90-deg plies, and these grow in density with the applied load. Similar damage modes for cross-ply laminates have been reported in the literature.[7,8] The visible damage initiates at lower load levels compared to the other laminates. Ultimate failure of these laminates is caused by fiber failure in the 0-deg plies.

To broadly summarize the experimental observations, when a specimen containing a crack is subjected to increasing tensile loads, the following sequence of events is observed.

1) At very low loads, the specimen deforms elastically with no damage.

2) At a threshold value of load, subcracks form parallel to the fibers of each ply at the notch tip.

3) As the load increases above the threshold value, the subcracks extend, and some region of delamination may form between plies.

4) At some critical maximum load, the fibers in the 0-deg plies of the laminate fail, causing the specimen to fail.

The laminate failure processes observed in this study can be separated into two relatively distinct categories. The first is general stress relief. Damage formation ahead of the crack tip in the form of matrix cracks relieves some portion of the high stress concentrated at the crack tip. Axial splits in the 0-deg plies are particularly effective in this regard, because they help reduce the stress concentration ahead of the tip in these plies.[8,10] Laminates with more extensive damage zones achieve more stress relief and are, therefore, less notch sensitive. Mandell et al.[9] have reported that these matrix cracks grow in length proportional to K^2 (stress intensity factor). The second category is ultimate failure caused by fiber failure of the principal load carrying plies (0-deg plies). X-ray and fractographic examinations reveal that these are the only plies that exhibit fiber failure, whereas all other off-axis plies fail along matrix cracks. Because fiber strength of a laminate is significantly higher than matrix strength, it is surmised that this is the parameter that governs the notched strength of the laminate.

Determination of Laminate Fracture Toughness

Notched strength in metals is characterized by a critical stress intensity factor or equivalently by a critical strain energy release rate. These are material parameters that uniquely characterize the fracture resistance of metals in the presence of cracks and are independent of the size of the crack. To study whether a similar parameter existed for notched composite laminates, the effect of crack length on critical failure stress was investigated in the study.

Laminate fracture toughness K_Q was defined as the value of the linear elastic stress intensity factor at the maximum test load. (This corresponded to catastrophic failure of the center-cracked tension specimens.) Fracture toughness of the laminate K_Q (or critical stress intensity factor) is calculated as

$$K_Q = Y \sigma_f \sqrt{\pi a} \tag{1}$$

where σ_f is the remote applied stress at failure, a is half the crack length, and Y is the correction factor to account for the finite width of the specimen, given by

$$Y = 1 + 0.1282(2a/W) - 0.2881(2a/W)^2 + 1.5254(2a/W)^3 \tag{2}$$

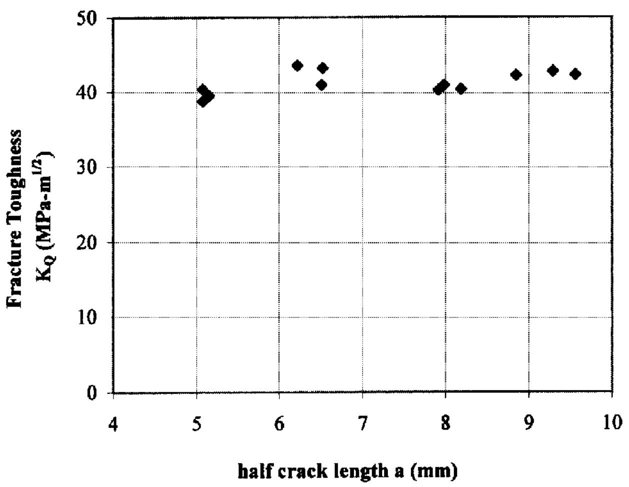

Fig. 8 Effect of varying crack length on K_Q for a $[90/0/45/-45]_s$ laminate.

Note that this is the correction factor originally developed for isotropic materials. However, researchers have reported[5,11] that it can still be used for orthotropic materials with little loss of accuracy. Also, the crack length is not adjusted by the characteristic damage zone size.

The results for a $[90/0/45/-45]_s$ laminate are shown in Fig. 8. They clearly indicate that K_Q is independent of crack length and can be considered a laminate material property. Similar results are observed for all of the other laminate configurations as well. The measured K_Q values for the nine laminates are presented in Table 2.

Fracture Criterion

As discussed in the preceding section, the fracture resistance for any laminate can be characterized by a unique fracture toughness parameter K_Q. Its value, of course, depends on laminate configuration. Most of the existing models[1–4] are limited to determining the value for K_Q experimentally by adjusting the damage zone size a_0. This is a major limitation, since experimental data for each laminate configuration are required to determine its damage zone size. The present research objective is to establish a more general parameter that is independent of laminate configuration.

Experimental results indicate that whereas matrix cracks provide local stress relief, final fracture of the laminate is controlled by fiber breakage initiating from the crack tip in the 0-deg plies. The entire laminate fails when the 0-deg plies within it fail. Fiber breakage of these plies, thus, can be considered the principal failure mechanism. This seems quite logical considering that fiber strength for composites is significantly higher than matrix strength. Previous researchers[6,7] studying cross-ply laminates have also shown that the 0-deg ply governs laminate notched strength.

It is proposed that the fracture toughness of the 0-deg ply (defined as K_Q^0) is a constant at failure, i.e., the laminate fails whenever the load in the 0-deg plies reaches the critical value K_Q^0. The existence of such a parameter for cross-ply laminates was first postulated by Kageyama,[7] who estimated its value by three-dimensional finite element analysis. This value corresponds to the toughness of the 0-deg ply when there is fiber breakage (Fig. 9). Unfortunately, this failure mode is not exhibited by unidirectional laminates, which fail due to axial splitting (matrix cracks parallel to fiber direction). Thus, K_Q^0 cannot be directly determined from experiments on notched unidirectional laminates. The parameter K_Q^0 for a unidirectional lamina within a laminate is an in situ or effective material parameter.

We can estimate the value of this parameter by simple stress analysis using lamination theory to calculate the portion of the applied load that is carried by the 0-deg plies in the laminate. Such an analysis is approximate, because it does not take into account any stress redistribution caused by local damage in the form of matrix cracks, etc. The effect of any such damage is lumped in the parameter K_Q. We define

$$\sigma_f^0 = \eta \sigma_f \tag{3}$$

where σ_f is the laminate applied stress at failure, σ_f^0 is the remote (far-field) stress in the 0-deg ply, and η is the factor relating the two stresses (η depends on the laminate configuration and material elastic constants and can be calculated using classical lamination theory).

Using Eq. (1) in Eq. (3), we obtain

$$\sigma_f^0 = \eta (K_Q / Y \sqrt{\pi a}) \tag{4}$$

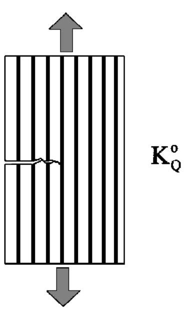

Fig. 9 Fracture toughness K_Q^0 corresponding to fiber breakage in the 0-deg ply.

or

$$Y\sigma_f^0\sqrt{\pi a} = \eta K_Q \tag{5}$$

We denote the left-hand side of Eq. (5) as K_Q^0. Thus,

$$K_Q^0 = \eta K_Q \tag{6}$$

Model Verification

According to the failure criterion in Eq. (6), the laminate fails when the load carried by its 0-deg plies reaches the critical value K_Q^0. It is a material constant and, thus, should be independent of the laminate orientation. To verify this hypothesis, the value of K_Q^0 within the different laminate configurations is calculated using Eq. (6) and plotted in Fig. 10 (see Table 2). The results indicate that the value of K_Q^0 is approximately the same in all of the laminates and does not deviate by more than 10% from the mean value of 110 MPa-m$^{1/2}$. Assuming this constant value for K_Q^0, the experimental results are compared with model predictions using Eq. (6) in Fig. 11. Good agreement is observed between the experimental data and model prediction.

The same model is used to predict the experimental results of other published researchers[12] (Figs. 12 and 13). The model works well for their data as well, though additional experimental data with different laminate stress ratios η is probably required for a better comparison.

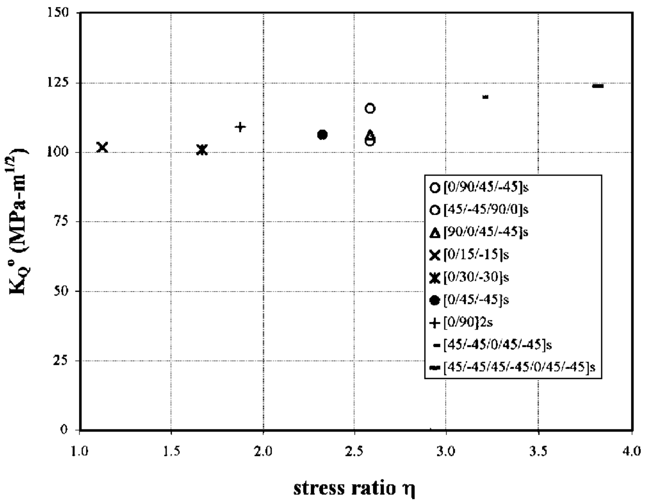

Fig. 10 Value of parameter K_Q^0 in different laminates.

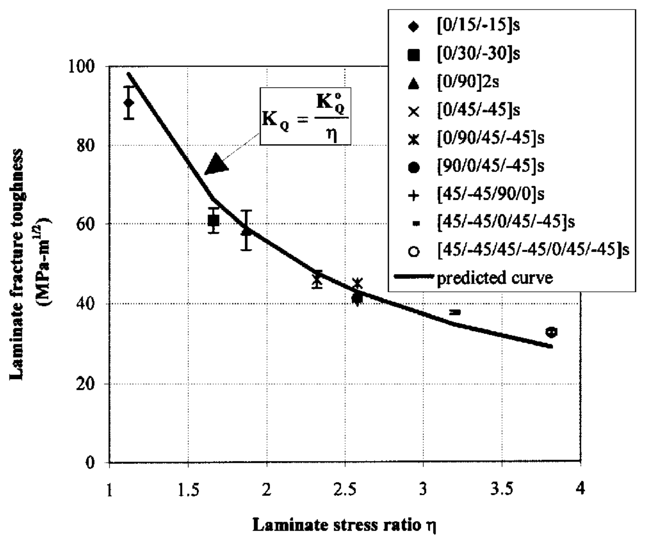

Fig. 11 Comparison of experimental results with model predictions for AS4/3501-6 laminates.

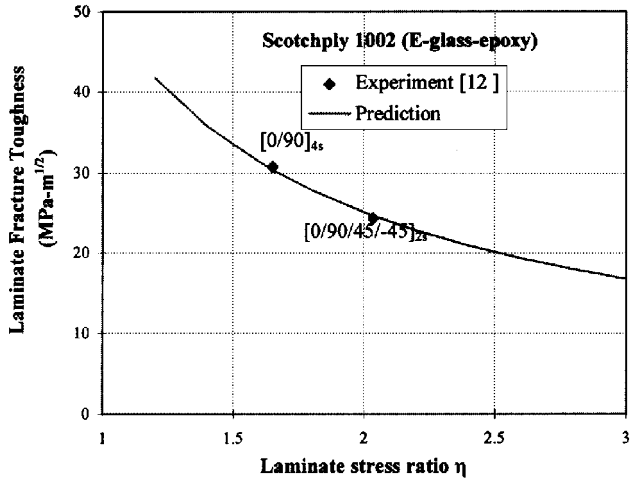

Fig. 12 Comparison of experimental results with model predictions for E-glass-epoxy laminates.

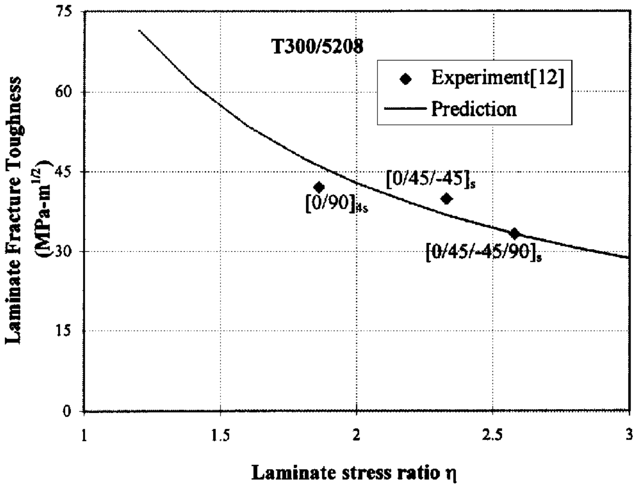

Fig. 13 Comparison of experimental results with model predictions for [T300/5708] laminates.

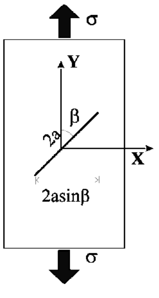

Fig. 14 Inclined crack configuration.

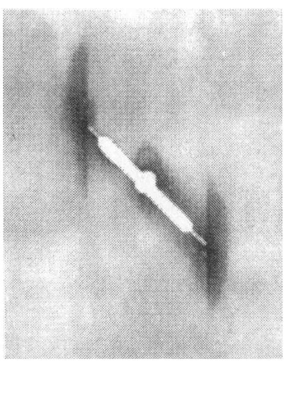

Fig. 15 Damage growth in a $[0/90/\pm 45]_s$ laminate with a crack at a 45-deg angle.

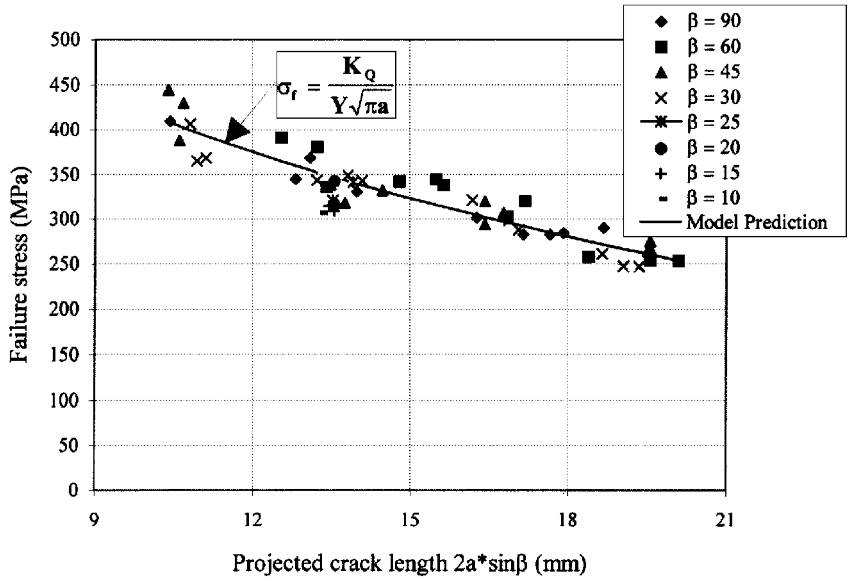

Fig. 16 Failure stress for inclined crack in $[0/90/\pm 45]_s$ laminates.

Thus, we can use a single parameter model to characterize the fracture behavior of these laminates. Preliminary tests are required for a single laminate configuration to determine K_Q^0, and once its value is determined, the fracture toughness of other laminates can be predicted.

As noted earlier in this section, the stress ratio η for a laminate is calculated using an elastic analysis. It does not account for stress redistribution in the crack tip region due to subcritical damage. The present model lumps the effects of damage induced stress relaxation into the fracture toughness parameter K_Q, and any significant redistribution due to a large damage zone would be reflected in a higher value for K_Q. The fact that such a model reliably predicts the fracture toughness of different laminates seems to indicate that the effect of damage induced stress relaxation is relatively small and is inherently accounted for within the model. Of course, if a very large damage zone exists (as is the case with cross-ply laminates with similar plies lumped together, e.g., $[0_2/90_2]_s$, $[90_3/0_3/90]_s$, etc.) a substantial redistribution takes place and the fracture toughness is elevated. The damage zone effect needs to be explicitly analyzed for such laminates and is a subject of our ongoing research.

Residual Strength of Laminates with Inclined Cracks

In most real-world situations, a crack is rarely oriented perpendicular to the loading direction. A more general scenario is that of a crack with an orientation arbitrary to the loading direction (Fig. 14). This problem is investigated to determine whether the present model can predict residual strength of laminates with inclined cracks. Attention is restricted to the case where the loading is along the 0-deg fiber direction (i.e., the material axis coincides with the loading axis). Center cracks were cut in the test panels at various angles (90-deg angle corresponds to the normal crack orientation discussed in the preceding section).

A radiograph of the crack tip damage in an inclined crack is shown in Fig. 15. The crack tip damage pattern in the form of axial splitting and matrix cracking is similar to that in a normal ($\beta = 90$ deg) crack in the same laminate. The eventual failure mode of the laminate for the two cases is also similar, with the off-axis plies failing along matrix cracks and the 0-deg plies failing by fiber fracture. The failure seems to be controlled by the stress concentration in the 0-deg ply near the crack tip. The relevant crack length parameter seems to be projection normal to the applied load (see Fig. 12).

Test results (Figs. 16 and 17) support this hypothesis. The residual strength of the laminates with different crack orientations β are plotted against their respective projected crack lengths. It is clear that results for different β values collapse onto a single curve that corresponds to the residual strength vs crack length curve for a

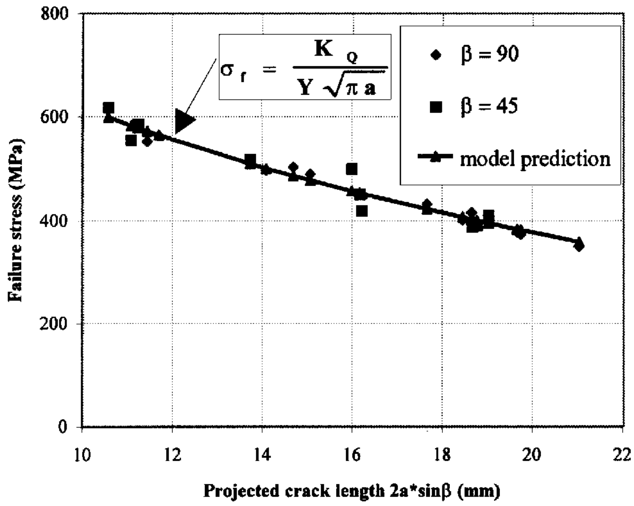

Fig. 17 Failure stress vs projected crack length for $[0/90]_{2s}$ laminate.

normal crack ($\beta = 90$). This master curve can be predicted by the model equation (6) discussed in the preceding sections. The failure stress for any orientation of the center crack in the $[0/90/\pm 45]_s$ laminate is, thus, given by

$$\sigma_f = \frac{K_Q^0/\eta}{Y\sqrt{\pi a_{\text{eff}}}} \tag{7}$$

where

σ_f = applied stress at failure
K_Q^0 = fracture toughness of the 0-deg ply
η = stress ratio for the laminate
a_{eff} = effective half-crack length, $a \sin\beta$
Y = finite width correction factor, $Y(2a_{\text{eff}}/W)$

The problem of predicting notched strength of such laminates with inclined cracks thus simplifies to the problem of a normal crack with effective crack length equal to the projection of the crack normal to the applied load.

Conclusions

A new failure criterion was proposed to predict the strength of composite laminates containing cracks. This criterion is based on the experimental observation that 0-deg plies in a laminate govern its strength. The fracture toughness of such 0-deg plies in the presence of fiber breakage was introduced as the material parameter K_Q^0, which is an in situ or effective material parameter. The parameter is layup independent and can be used to predict the notched strength/fracture toughness of any laminate configuration that contains some 0-deg plies. The comparison with experimental data indicates that the model provides a good estimate of laminate notched strength for different configurations. The model was also extended to predict notched strength of laminates with arbitrary crack orientations. In such cases the projection of the crack normal to the applied load can be considered as the effective crack length.

Acknowledgments

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